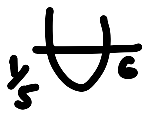
Chapter 1 to 5 test

1. (a) Find the range of values of x satisfying the inequality $(5x - 1)(6 - x) < 0$.

$\frac{1}{5}$ $\quad 6$

[2]



$$\frac{1}{5} < x < 6$$

(b)Find the values of k such that the line $y = 9kx + 1$ does not meet the curve $y = kx^2 + 3x(2k + 1) + 4.$

[5]

$$kx^2 + 3x(2k+1) + 4 = 9kx + 1$$

$$kx^2 + \underline{6kx} + 3x + 4 - \underline{9kx} - 1 = 0$$

$$a = k, \quad b = -3k + 3, \quad c = 3$$

$$b^2 - 4ac < 0$$

$$(-3k+3)^2 - 4(k)(3) < 0$$

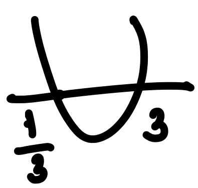
$$9k^2 - 18k + 9 - 12k < 0$$

$$9k^2 - 30k + 9 < 0$$

$\div 3$

$$3k^2 - 10k + 3 < 0$$

$$(k-3)(3k-1) < 0$$



$$\frac{1}{3} < k < 3$$

2. Find constants a, b and c such that $\dfrac{\sqrt{p}q^{\frac{2}{3}}r^{-3}}{(pq^{-1})^2 r^{-1}} = p^a q^b r^c$.

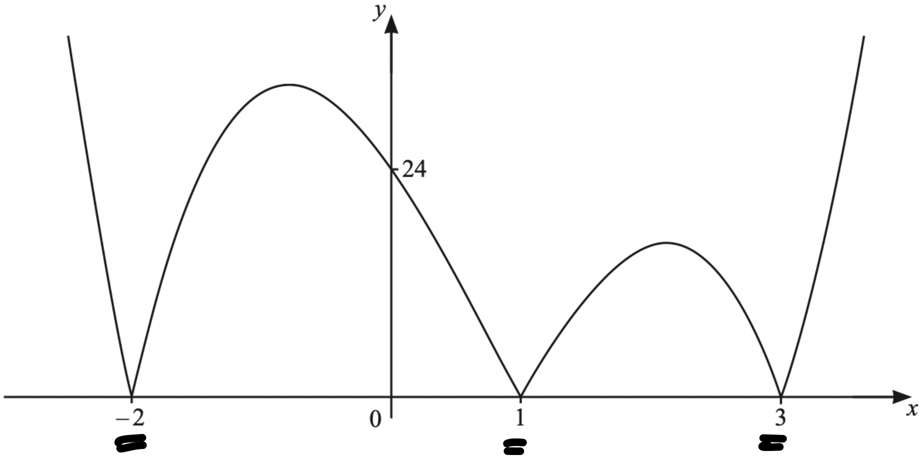
[3]

$$\frac{p^{1/2} q^{2/3} r^{-3}}{p^2 q^{-2} r^{-1}}$$

$$= p^{-3/2} q^{8/3} r^{-2}$$

$a = -\frac{3}{2}$, $b = \frac{8}{3}$, $c = -2$

3. (a)



The diagram shows the graph of $y = |f(x)|$, where f(x) is a cubic. Find the possible expressions for f(x).

$f(x) = \pm 4(x+2)(x-1)(x-3)$

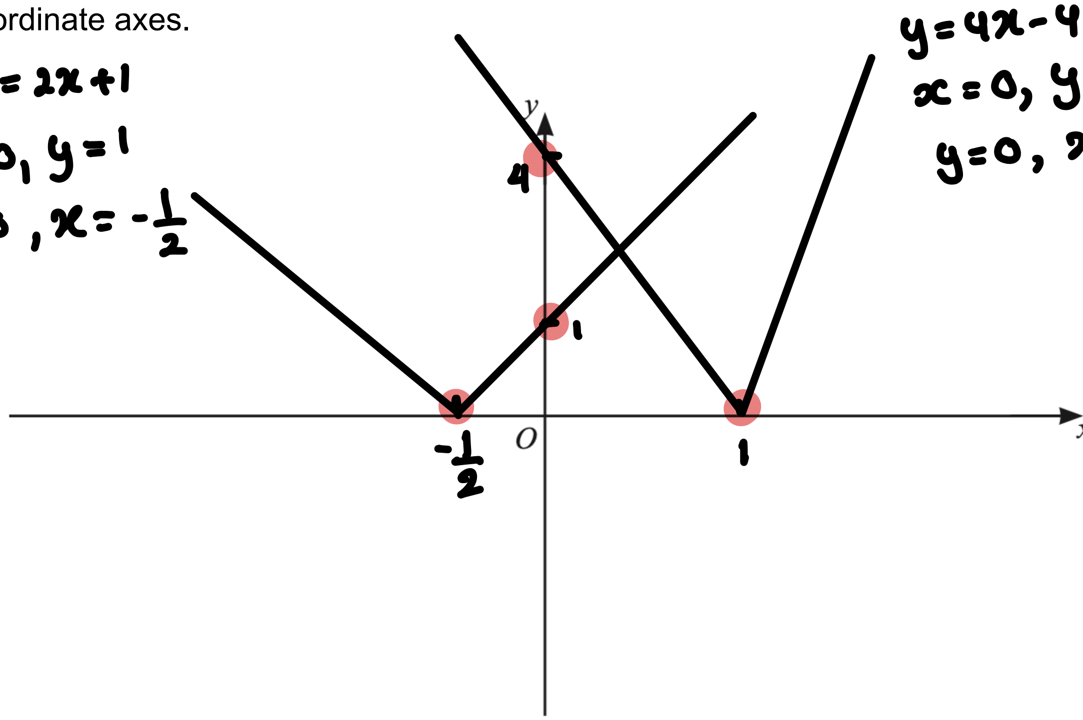
[3]

(b) (i) On the axes below, sketch the graph of $y = |2x + 1|$ and the graph of $y = |4(x - 1)|$, stating the coordinates of the points where the graphs meet the coordinate axes.

$y = 2x+1$

$x=0,\ y=1$

$y=0,\ x=-\frac{1}{2}$



$y = 4x-4$

$x=0,\ y=-4$

$y=0,\ x=1$

[3]

(ii) Find the exact solutions of the equation $|2x + 1| = |4(x - 1)|$.

[4]

$(2x+1)^2 = (4x-4)^2$

$4x^2 + 4x + 1 = 16x^2 - 32x + 16$

$0 = 12x^2 - 36x + 15$

$\div 3$

$0 = 4x^2 - 12x + 5$

$0 = (2x-5)(2x-1)$

$2x - 5 = 0$ or $2x - 1 = 0$

$x = \frac{5}{2}$

$2x = 1$

$x = \frac{1}{2}$

4. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Find the exact coordinates of the points of intersection of the curve $y = x^2 + 2\sqrt{5}x - 20$ and the line $y = 3\sqrt{5}x + 10$.

[4]

$$3\sqrt{5}x + 10 = x^2 + 2\sqrt{5}x - 20$$

$$0 = x^2 - \sqrt{5}x - 30$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{\sqrt{5} \pm \sqrt{5 - 4(1)(-30)}}{2}$$

$$= \frac{\sqrt{5} \pm \sqrt{5 + 120}}{2}$$

$$= \frac{\sqrt{5} \pm \sqrt{125}}{2}$$

$$= \frac{\sqrt{5} \pm 5\sqrt{5}}{2}$$

$$= \frac{\sqrt{5} + 5\sqrt{5}}{2} \quad \text{or} \quad \frac{\sqrt{5} - 5\sqrt{5}}{2}$$

$$= 3\sqrt{5} \quad \text{(or)} \quad -2\sqrt{5}$$

$$y = 3\sqrt{5}x + 10 \qquad y = -30 + 10$$

$$= 45 + 10 \qquad = -20$$

$$= 55$$

5. The polynomial $p(x) = mx^3 - 17x^2 + nx + 6$ has a factor $x - 3$. It has a remainder of -12 when divided by $x + 1$. Find the remainder when $p(x)$ is divided by $x - 2$.

[6]

$p(3) = 27m - 153 + 3n + 6$

$0 = 27m + 3n - 147$

$27m + 3n = 147$

$\div 3$

$9m + n = 49$ — ①

$p(-1) = -m - 17 - n + 6$

$-12 = -m - n - 11$

$m + n = 1$ — ②

$9m + n = 49$

$-8m = -48$

$m = 6$

$m + n = 1$

$6 + n = 1$

$n = -5$

$p(x) = 6x^3 - 17x^2 - 5x + 6$

$p(2) = -24$

6. (a) Write $9x^2 - 12x + 5$ in the form $p(x - q)^2 + r$, where p, q and r are constants.

$p(x^2 - 2qx + q^2) + r$

$px^2 - 2pqx + pq^2 + r$

[3]

$p = 9 \quad -2pq = -12$

$-18q = -12$

$q = \frac{4}{6} = \frac{2}{3}$

$pq^2 + r = 5$

$9(2/3)^2 + r = 5$

$4 + r = 5$

$r = 1$

$\therefore 9(x - 2/3)^2 + 1$

(b) Hence write down the coordinates of the minimum point of the curve $y = 9x^2 - 12x + 5$.

$(2/3, 1)$

[2]

7. Find the value of x such that $\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$.

$$\frac{2^{2x+2}}{2^{x-1}} = 2^{5x/3} \times 2^1$$

[5]

$$2^{2x+2-x+1} = 2^{\frac{5x}{3}+1}$$

$$x + 3 = \frac{5}{3}x + 1$$

$$-\frac{2}{3}x = -2$$

$$x = 2 \times \frac{3}{2} = 3$$

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